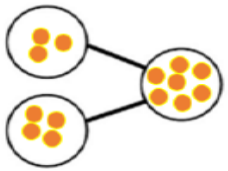
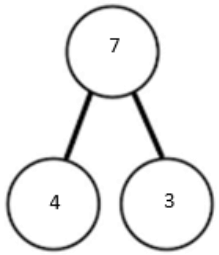
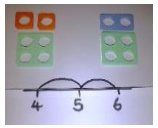
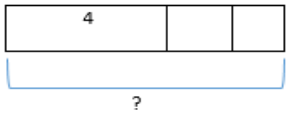
Addition

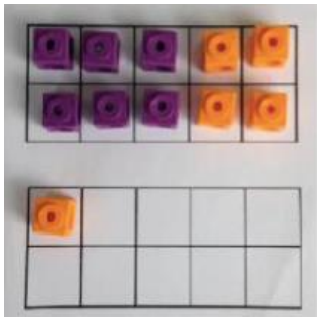
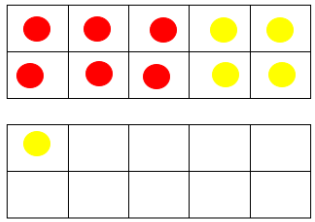
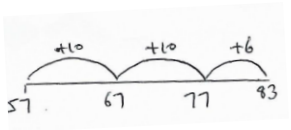
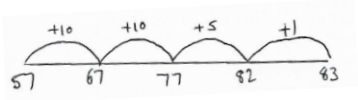
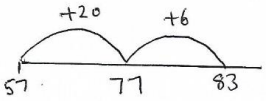
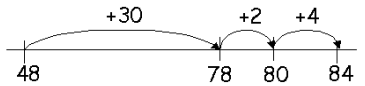
The table below details the stages that pupils go through in their learning of addition, culminating in them carrying out a formal written addition method with fluency, accuracy and understanding. The aim is that pupils can identify addition calculations for which a mental method is appropriate, but for calculations that they cannot do in their heads they choose an appropriate written method.

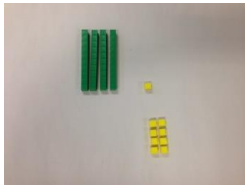
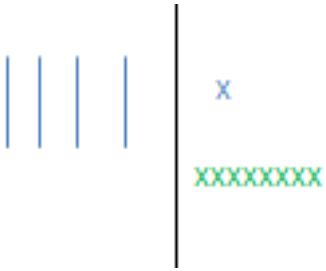
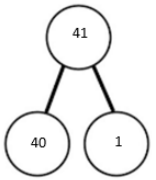
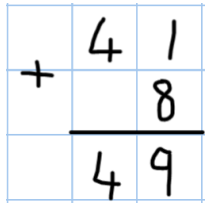
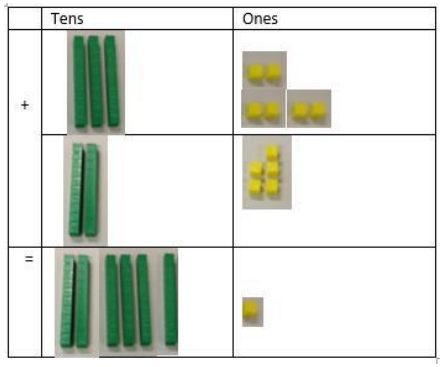
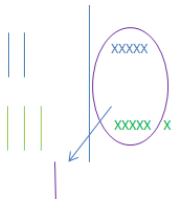
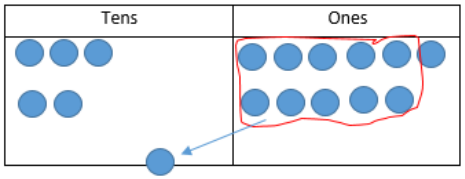
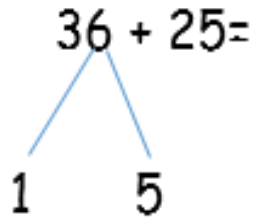
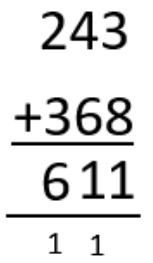
Time must be taken building up to the formal written method to ensure complete understanding at each stage.

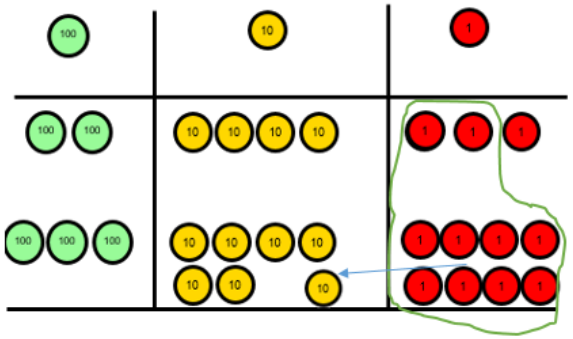
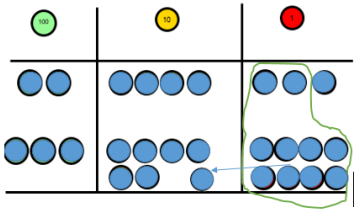
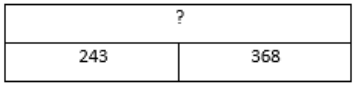
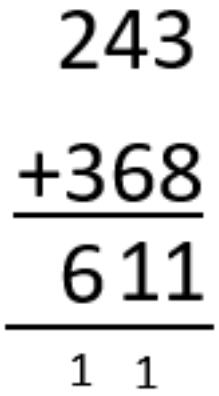
Each Stage has a suggested year group during which the particular Stage will be taught. It will not be the case that all pupils in a year group are working at that year group's stage; however, the majority of pupils within a year group will be. Some pupils will need extra time to consolidate their understanding of a stage prior to stage within their own year group.

Within stages, pupils' learning should start within the Concrete step of the stage (if this is present), with pupils using practical equipment to solve calculations. Learning progresses to the Pictorial step, where pupils draw what they have created in the Concrete step. Finally, pupils should move to the Abstract step. At each Stage, a deep understanding (or mastery) of the stage should be demonstrated by pupils. This will be evident through them being able to use the correct method when a calculation is presented in a range of different formats or using varied language, and through pupils explaining the steps involved in the specific calculation method that they are using.

Year	Stage	Concrete	Pictorial	Abstract
1	**1 - Combining two parts to make a whole**	Pupils use multilink cubes and other resources such as small plastic toys etc. Eg, 3 + 4 =  combined to make 	It is important for pupils to represent this in a variety of ways, for example with 4 at the top and 3 at the bottom, or the whole (7) on the left and the parts (3 and 4) on the right. 	3 + 4 = 4 is a part, 3 is a part and the whole is 7. 
1	**2 - Counting on using number lines**	Pupils use practical equipment such as Numicon. Starting at the larger number on a number line and counting on from it. Eg, 4 + 2  	A bar model which leads pupils to count on. 	Use of a number line drawn by pupils. The addition calculation is carried out without the use of practical equipment. Variety of language and questioning, eg. What is 2 more than 4? What is the sum of 4 and 2? What is the total of 4 and 2? What is 2 plus 4? 
1	**3 - Regrouping**	This stage is closely related to, and dependent on, pupils acquiring fluency and mastery of	Pictorial representation of regrouping to make 10.	Pupils to develop understanding of equality, and the = sign meaning

	to make 10.	number bonds to 10. Pupils use cubes and/or Numicon to regroup the calculation to make 10. Eg, 6 + 5 =  or 	6 + 5 = Pupils draw counters or cubes and regroup the calculation to make 10 then add on the counter(s) remaining. 	'equal' and 'the same'. Eg 6 + ☐ = 11 And 6 + 5 = 5 + ☐ This type of question then progresses to questions such as: 6 + 5 = ☐ + 4
1	**4 – counting on in 10s, multiplies of 10 and other numbers using a number line and**	Children should be able to count on using a numbered number line, supported when necessary with practical equipment such as multilink. Progressing from Stage 2 above, they use their fluency in regrouping developed during Stage 3 to make logical jumps based on 10, and other numbers if appropriate. Each use of the number line begins with the larger number. Eg 26 + 57 = 57 + 10 + 10 + 5 + 1 =  Or 57 + 10 + 10 + 6 = 	This stage is the same as the Concrete step but pupils do not have the support of practical equipment such as multilink. Furthermore, jumps on the number line should be progressing to be in multiples of 10 rather than 10. 	Pupils draw their own number lines to complete calculations. The empty number line helps to record the steps on the way to calculating the total. The steps often bridge through a multiple of ten. Allow children to experiment with the order of adding to allow them to understand that addition can be done in any order. Eventually refine this to starting with the largest number, adding the tens and then adding the ones. e.g. 48 + 36 = 84 

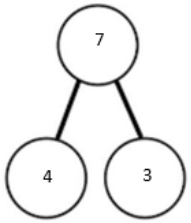
2	**5 – Column method no regrouping**	T1s + 1s using Base 10. Pupils continue to develop fluency in partitioning and mastery of place value. Eg, 41 + 8 = 	Pupils progress to drawing the concrete Base 10 equipment using symbols. Introduction of place value columns. 10s 1s 	Pupils partition the two digit number:  Then add 1s (so 1 + 8 = 9), before regrouping with the 10s (40 + 9 = 49). Following this, formal column method (which relies on mastery of use of place value columns introduced in the Pictorial step). 
3	**6 – Column method to 2 digit + 2 digit.**	Pupils need to continue to develop their understanding of partitioning and place value, in particular the use of place value columns. Use of Base 10 equipment at this step. Eg, 36 + 25 = 	Pupils progress as in Stage 5 above to drawing the Base 10 equipment. This drawing is either: 10s 1s  Or: 	Progression from drawings. Firstly, partitioning based on mastery of number bonds to 10. Pupils look for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Following this, formal column method: 

4	**7 – HT1s + HT1s.**	Use of place value counters (or Base 10 equipment) to add HT1s + T1s, HT1s + HT1s etc. Pupils need to be given the opportunity to continue to develop their knowledge of the place value columns, extending into the thousands column and beyond if appropriate. Once pupils have developed fluency with this, they need to progress to applying their understanding to larger numbers. Eg, 243 + 368 = 	Pupils to represent the counters or Base 10 equipment using drawings:  If pupils are completing a word problem they are to draw a bar model to represent what they are being asked to do:  	Use of the formal written method. 
4, 5, 6	**8 – Compact column method**			This is the formal standard method of addition. This method should be taught when children are completely confident in using the expanded column method and can prove this by using Base 10 equipment in the previous Stage. This expanded method taught in previous Stages is now shortened: when the column total is a two digit number, the tens (or hundreds) are carried over to the next column. Use the words “carry ten” or “carry one hundred” **not** “carry one”.

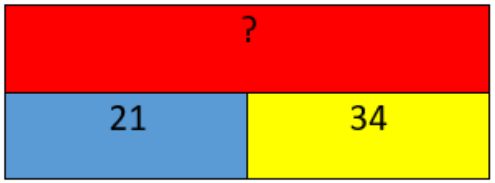
				e.g. 789 + 642 = 1 431 Th H T 1s 7 8 9 6 4 2 1 4 3 1 1 1 1 1 Within this Stage, pupils should progress through the following, using the compact method: 1. Addition of two, three and four digit numbers. (Two or more numbers). 2. Addition of decimal numbers to 1, 2 and 3 decimal places. (Including amounts of money and other measures). 3. Addition of decimal numbers where the two numbers have a different number of decimal places e.g. 1.78 + 54.2 =.

Developing fluency and mastery of addition through variation

Pupils need to master addition as they progress through the Stages detailed above. This is achieved through pupils becoming comfortable in presenting their working out in a variety of formats, such as:



Or:



Pupils need to be able to explain why they have selected a particular way of presenting their working out.

Variation is also achieved through varying the language and words which are used to present calculations and questions:

Sam saved £21 one week and £34 in another. How much did he save **in total**? (See use of a bar model above to represent this).

Questions need to be presented in a variety of ways so that pupils develop their understanding of, for example, the = sign meaning 'equals' rather than 'makes'.
Eg,
21 + 34 = ☐

Presented as:

☐ = 21 + 34

What is the **sum** of 21 and 34?

Key words which pupils need to become used to hearing and seeing when learning addition:
Add, more, and, make, altogether, total, equals, equality.
Plus, sum, addition, partition, count on, tens boundary.
Altogether.
Hundreds boundary, increase, exchange, carry.
Plus, sum, addition, partition, count on, tens boundary, hundreds boundary, increase, exchange, carry, decimal, decimal point, tenths, hundredths, thousandths, inverse.

Prove it and explain questions

33 + 26 = 59. How can you prove this? Questions such as this challenges pupils to show an understanding of subtraction as being the inverse of addition.

41 + 16 = 35. Explain how you know that this is wrong. Questions such as this require pupils to understand the nature of addition: specifically that addition of positive numbers increases value.