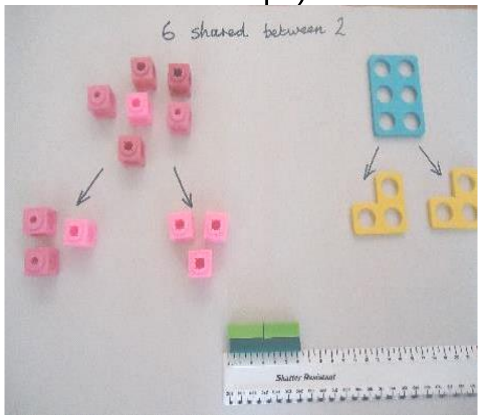
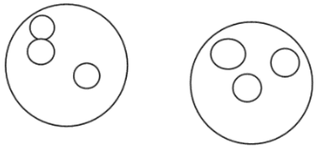
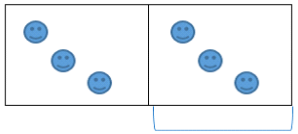
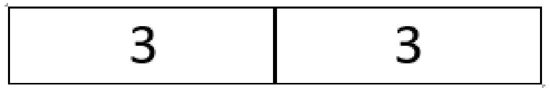
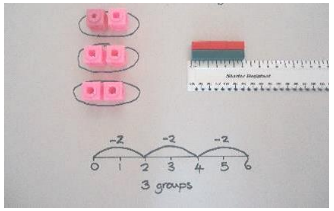
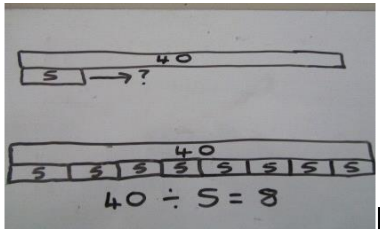
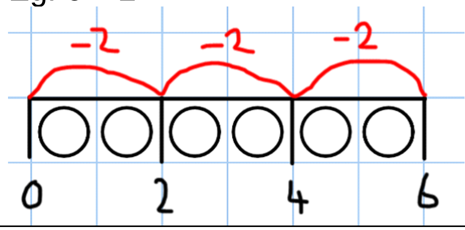
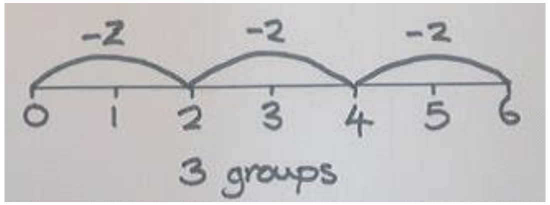
Division

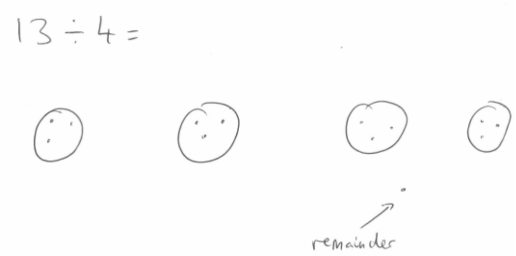
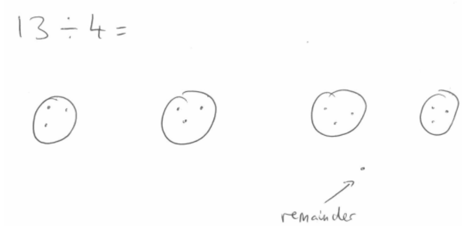
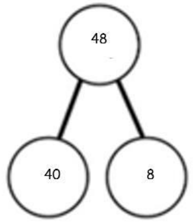
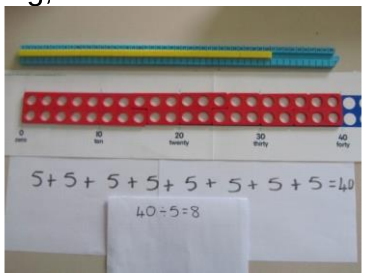
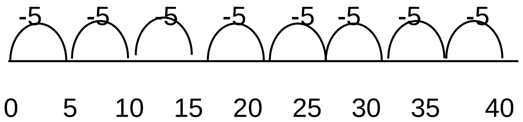
The table below details the stages that pupils go through in their learning of division, culminating in them carrying out a formal written division method with fluency, accuracy and understanding. The aim is that pupils can identify division calculations for which a mental method is appropriate, but for calculations that they cannot do in their heads they choose an appropriate written method.

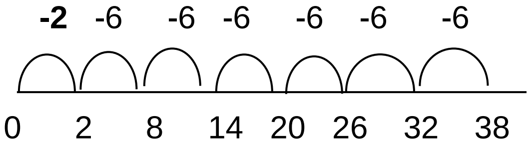
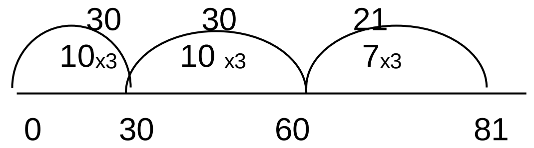
Time must be taken building up to the formal written method to ensure complete understanding at each stage.

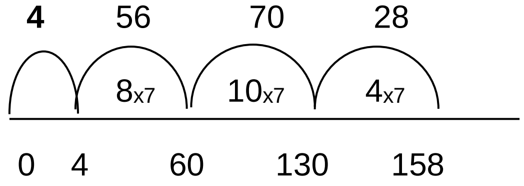
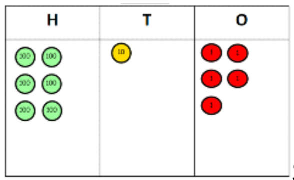
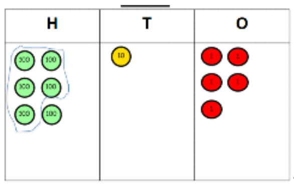
Within stages, pupils' learning should start within the Concrete step of the stage (if this is present), with pupils using practical equipment to solve calculations. Learning progresses to the Pictorial step, where pupils draw what they have created in the Concrete step. Finally, pupils should move to the Abstract step. At each stage, a deep understanding (or mastery) of the stage should be demonstrated by pupils. This will be evident through them being able to use the correct method when a calculation is presented in a range of different formats or using varied language, and through pupils explaining the steps involved in the specific calculation method that they are using.

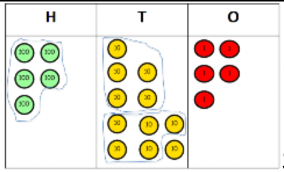
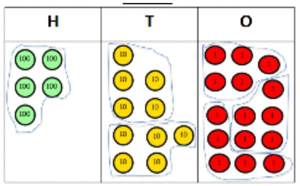
Year	**Stage**	**Concrete**	**Pictorial**	**Abstract**
1	**1 – sharing and grouping (practical division).**	6 shared between 2 (a range of practical objects can be used – eg children themselves in hoops).  6 ÷ 2 =	6 ÷ 2 = Pupils draw the practical division:  The use of a bar model here is important, so that all 4 operations are shown using bar models. 	6 ÷ 2 = What is the calculation? Use of bar model now progresses from pictures being drawn on the bar to numbers: 

1 and 2	**2 – division as repeated grouping and repeated subtraction.**	Although this Concrete step is very closely related to the Concrete step of stage 1, the language used to describe the step here needs to focus on use of the word **repeated**, to emphasise the repeated nature of division. Eg: 6 ÷ 2 = 	Use of a bar model which pupils draw: Eg: 40 ÷ 5 = How many lots of 5 equal 40? How many times must we **repeat** the drawing of 5 to get up 40? Progressing from the bar model to a number line which pupils draw, including pictorially representations of practical equipment: Eg: 6 ÷ 2 = 	Use of the abstract number line, drawn by pupils, without the use of drawing of individual counters: Eg: 6 ÷ 2 = 
3	**3 – introduction of remainders**	Eg, 13 ÷ 4 = Use of a range of practical equipment. Discussion around the one piece of	Pupils draw what they have done in the Concrete step, once more focussing discussion	Pupils partition the starting number of the number sentence before calculating the division of each

	and reinforcement of division as sharing using practical equipment.	equipment which ‘cannot’ be put into a group. (This drawing should be done using practical equipment).  How many pieces in each group? 3. How many pieces cannot be put in a group? 1. So, our answer is 3 remainder 1.	around the one piece of equipment (one dot etc.) which ‘cannot’ be put into a group. Eg, 13 ÷ 4 = 	partitioned element of that number: Eg, 48 ÷ 4 =  4 tens ÷ 4= 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12
3	**4 – number lines.**	A number line created using practical equipment such as Numicon. Eg, 40 ÷ 5 = 	Pupils to complete the same process as in the Concrete step, but drawn models onto a pre-drawn number line.	Pupils to draw their own number lines to show division. Eg, 40 ÷ 5 =  How many lots of 5 did we need to get to 0? 8, so the answer is 8.

		How many 5s are there in 40? The 5 Numicon piece is used to fill up a number line until 40. How many pieces of 5 did you need? 8, so the answer is 8.		<u>Example with remainder</u> Eg, 38 ÷ 6 = 6 r 2  Fractions of quantities (where the numerator is 1 and the denominator is under 12) should be introduced alongside division. (For example, children can find one fifth of 40 and realise that this is the same as 40 ÷ 5). <u>Examples for a larger numbers</u> For larger numbers it would be inefficient to count in single multiples so bigger jumps need to be recorded using known facts. e.g without a remainder 81 ÷ 3 = 27  This is done by working out the numbers of threes in each jump as you go along (10 threes are 30, another 10 threes makes 60, and another 7 threes makes 81. That’s 27 threes altogether). e.g. with a remainder

				158 ÷ 7  So, 158 ÷ 7 = 22 r 4
4 and 5	**5 – use of place value in order to understand and develop of fluency in the bus stop method.**	Use of drawn place value grids and equipment such as multilink cubes. Eg, 615 ÷ 5 = Step 1: make 615. Step 2: Starting from the largest place value column, circle your groups of 5 in that column.	Pupils to represent the practical equipment they used in the Concrete step pictorially. This can also be done using place value columns which include decimal place value columns, when dealing with remainders.	Formal written bus stop method. **<u>No remainders</u>** 2 7 3 ⟌ $8^{2}1$ 81 ÷ 3 Using place value knowledge, pupils begin by considering ‘how many 3s in 80?’ (division fact combined with place value knowledge required). Pupils write 2 (which means ‘20’ because it is in the 10s column) above the 8, and write the remainder to the left of the 1s digit (link this to the exchanging process carried out in the Concrete and Pictorial steps). Then, ‘How many threes in 21?’. **<u>Calculations with remainders</u>** Eg, 284 ÷ 6 =

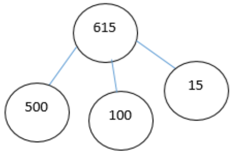
		Step 3: There is 1 hundred not in a group of 5, so exchange that hundred for 10 tens and circle groups of 5 in the 10s column. Step 4: There is now 1 ten not in a group of 5, so exchange that ten for 10 ones and circles groups of 5 in the 1s column. How many groups of 5 in each column? The answer is 1 hundred, 2 tens and 3 ones, so 123.		4 7 r 2 6 | 2 8 ^{4}4 **<u>Remainders converted into fractions</u>** 4 7 r 2 6 | 2 8 ^{4}4 Eg, 284 ÷ 6 = Discuss how the remainder 2 relates to the question being divided by 6, so that the 2 becomes the numerator at the end of the answer, and the 6 becomes the denominator. Pupils to demonstrate mastery of fractions by simplifying any fractions when possible. So, 284 ÷ 6 = 47 $^2/_6$ which is 47 $^1/_3$. **<u>Remainders converted into decimals</u>** Eg, 282 ÷ 4 = **7 0 . 5** **4 | 2 8 2 . ^{2}0 0**

5 and 6	**6 – use of the bus stop method when using long division.**	If felt necessary, the Concrete step of Stage 5 above can be used here.	If felt necessary, the Pictorial step of Stage 5 above can be used here.	This is a separate stage because multiplication knowledge generally cannot be used when answering these questions. **<u>With a remainder</u>** 2 8 r 12 15 | $4\ ^{4}3\ \ 2$ 3 0 1 3 2 1 2 0 1 2 432 ÷ 15 = 28 r12 **<u>Example with the remainder as a fraction</u>** 2 8 r 12 15 | $4\ ^{4}3\ \ 2$ 3 0 1 3 2 1 2 0 1 2 432 ÷ 15 = 28 $^{12}/_{15}$ Pupils should simplify $^{12}/_{15}$ to $^{4}/_{5}$. **<u>Example with the remainder as a decimal</u>** 2 8 . 8 15 | $4\ ^{4}3\ \ 2\ .0$ 3 0 1 3 2 1 2 0 1 2 0 1 2 0 0

				432 ÷ 15 = 28.8

Developing fluency and mastery of division through variation

Pupils need to master division as they progress through the stages detailed above. This is achieved through pupils becoming confident in answering the same division question when presented in a range of formats, including worded problems.
Eg, using 615 ÷ 5 to develop fluency and mastery of division.



Using this model, how can you divide 615 by 5 without using the bus stop method? (Divide each partitioned element of 615 by 5 then recombine by adding).

Worded problems:

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

Also, give pupils the number sentence and ask them to write a number story (ie worded problem) based on it.
Write a maths story involving 615 ÷ 5.

Present the question using different formats:

$5 \overline{)615}$

615 ÷ 5 =

= 615 ÷ 5

How many 5s go into 615?

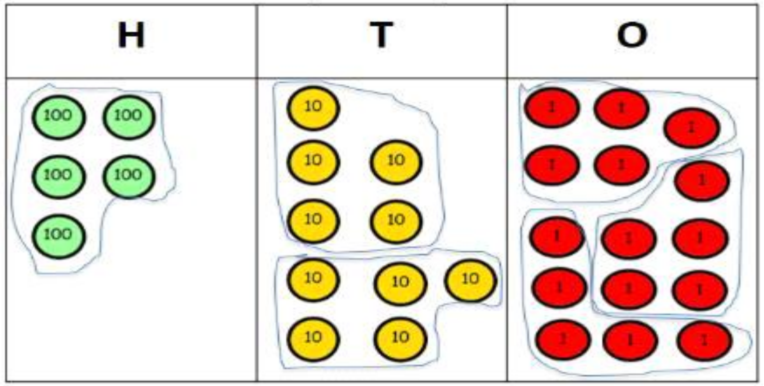
Prove it and explain questions

Prove using two different methods that 615 ÷ 5 = 123

Explain how you know that 615 ÷ 5 = 125 is wrong.

Present working out without giving the question and ask pupils to explain what the question is:

What's the question that this working out has solved? What's the answer?



Variation is also achieved through varying the language and words which are used to present calculations and questions:

Share, share equally, one each, two each, group, groups of, lots of, half, array, divide, division, fraction, inverse, remainder, quotient (the answer), divisor (the number you are dividing by), dividend (the number being divided), decimal.