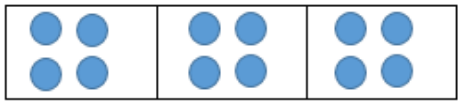
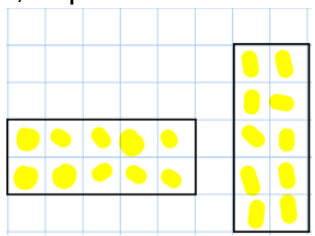
# Multiplication

The table below details the stages that pupils go through in their learning of multiplication, culminating in them carrying out a formal written multiplication method with fluency, accuracy and understanding. The aim is that pupils can identify multiplication calculations for which a mental method is appropriate, but for calculations that they cannot do in their heads they choose an appropriate written method.

Time must be taken building up to the formal written method to ensure complete understanding at each stage.

Each stage has a suggested year group during which the particular stage will be taught. It will not be the case that all pupils in a year group are working at that year group's stage; however, the majority of pupils within a year group will be. Some pupils will need extra time to consolidate their understanding of a stage prior to their own year group's.

Within stages, pupils' learning should start within the Concrete step of the stage (if this is present), with pupils using practical equipment to solve calculations. Learning progresses to the Pictorial step, where pupils draw what they have created in the Concrete step. Finally, pupils should move to the Abstract step. At each stage, a deep understanding (or mastery) of the stage should be demonstrated by pupils. This will be evident through them being able to use the correct method when a calculation is presented in a range of different formats or using varied language, and through pupils explaining the steps involved in the specific calculation method that they are using.

| Year | Stage | Concrete | Pictorial | Abstract |
|---|---|---|---|---|
| 1 | **1 – repeated group and repeated addition.** | Eg, 3 x 4 = or 3, 4 times. Pupils physically move and group cubes or other objects, and count up to find the answer.<br>Progress to counting up in the groups (so in this example, counting up in 4s).<br>  | Pupils to represent the Concrete step pictorially:<br>XX XX XX<br>XX XX XX<br><br>Develop use of a bar model for a more structured method of presentation:<br>  | 3 x 4 presented as 3, 4 times, or 4 added 3 times, in a written form:<br><br>4 + 4 + 4 = |
| 1 and 2 | **2 – use of arrays to learn commutativity.** | The commutative nature of multiplication is a crucial concept: it facilitates pupils learning their multiplication tables, and demonstrates mastery of the understanding what multiplication means.<br><br>Commutativity is 2 x 5 = 5 x 2. Put simply, the order of each number does not matter in a multiplication calculation. | Pupils to draw the arrays, presented as 5,2 times and 2,5 times, to prove commutativity:<br>  | Pupils use an array to write a range of calculations:<br><br>Eg, 2 x 5 = 10<br>5 x 2 = 10<br>2 + 2 + 2 + 2 + 2 = 10<br>5 + 5 = 10 |

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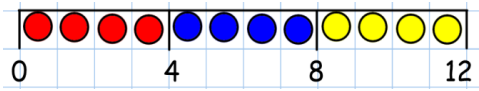
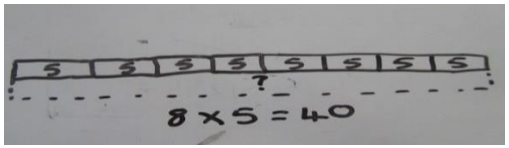
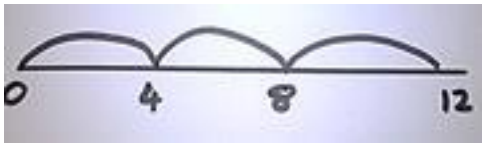
| | | Eg, 2 x 5 = 5 x 2.<br><br>At this step, physical counters or blocks are used to demonstrate this:<br>   <br>By counting up the blocks, pupils can see that 2, 5 times is the same as (equal to) 5, 2 times. | | |
| 2 and 3 | **3 – use of a number line to show repeated addition.** | Use of practical equipment such as Numicon, which are used alongside the number line:<br><br>Eg, 3 x 4 =<br>  <br>  | Number line represented pictorially, with range of colours used to show different groups.<br><br>Eg, 3 x 4 =<br>  | Pupils can start by using a bar model, rather than moving immediately to jumps.<br><br>Eg, 8 x 5 =<br>  <br>Then, a number line that pupils have created themselves, showing jumps rather than drawing of counters.<br><br>Eg, 3 x 4 =<br>  |

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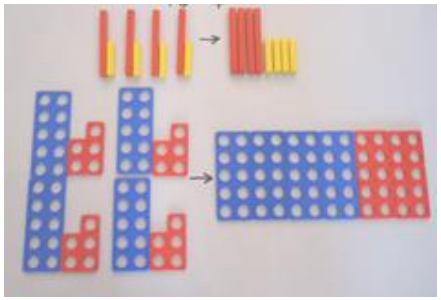
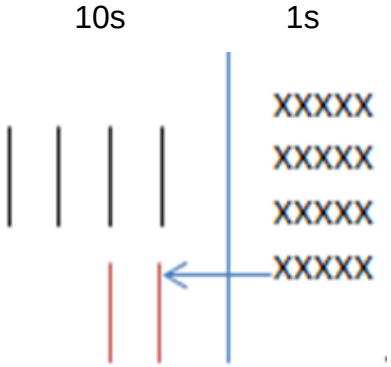
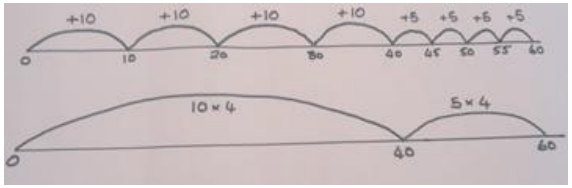
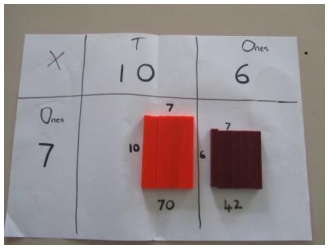
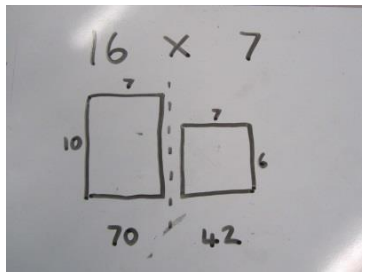
| 3 | **4 – use of partitioning to multiply.** | Using Numicon or Base Ten, pupils to partition one of the numbers in the calculation.<br>Pupils to develop an understanding of which number to partition.<br><br>Once partitioned, the practical equipment put together to show the answer.<br><br>Eg, 4 x 15 =<br>  | Pupils to represent the concrete equipment in pictures, using place value columns.<br><br>Eg, 4 x 15 =<br>  | Pupils to partition without the use of pictorial representation. At first, a number line is used.<br><br>Eg, 4 x 15 =<br>  <br>Then, pupils to use multiplication table knowledge to complete each partitioned element of the calculation.<br>4 x 15<br>10 5<br><br>10 x 4 = 40<br>5 x 4 = 20<br>40 + 20 = 60 |
| 3 | **5 – grid method** | Pupils use physical equipment within a grid. Understanding of partitioning, practised in Stage 4, is needed in this Stage.<br>Eg, 16 x 7 =<br>  | Pupils draw the physical equipment which they used in the concrete step.<br>Eg, 16 x 7 =<br>  | Pupils progress to presenting the concrete step using numbers.<br><br>Eg, 16 x 7 =<br><br>X \| 10 \| 6<br>7 \| 70 \| 42 \| 112<br><br>Ensure that pupils have practice multiplying numbers by 10, 100 and 1 000, to avoid them thinking that to do 10 x 7 they simply ‘add a zero’. |

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<u>Multiplying a 2 digit by a 2 digit number</u><br>
Pupils should partition both numbers and multiply each part. Children can then add the parts together, using column addition if needed:<br>
e.g. 56 x 27 =<br><br>
<table>
<tr><td>x</td><td>20</td><td>7</td><td>+</td></tr>
<tr><td>50</td><td>1 000</td><td>350</td><td>1 350</td></tr>
<tr><td>6</td><td>120</td><td>42</td><td>162</td></tr>
<tr><td></td><td></td><td></td><td>1 512</td></tr>
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<br>
<u>Multiplying by a decimal</u><br>
Eg, 36.8 x 7 =<br>
<table>
<tr><td>x</td><td>30</td><td>6</td><td>0.8</td><td>+</td></tr>
<tr><td>7</td><td>210</td><td>42</td><td>5.6</td><td>257.6</td></tr>
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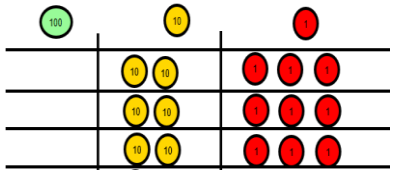
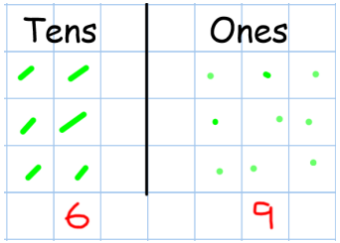
<td>3 and 4</td>
<td>6 – introduction of the column method.</td>
<td>Use of practical equipment such as place value counters of multilink.<br><br>At first, use calculations with no exchanging, before progressing to calculations with exchanging (this progression is an example of procedural variation).<br><br>Eg, 3 x 23. Make 23, 3 times. Using a place value grid and practical equipment, see how many ones and how many tens there are when pupils have laid out 23, 3 times.<br><br>  </td>
<td>Pupils to draw their own place value grids, and represent the practical equipment in a pictorial way.<br><br>Eg, 3 x 23 =<br><br> 
</td>
<td>Pupils to record the partitioning of the previous steps numerically.<br><br>3 x 23<br>20 3<br>Because of this partitioning:<br><br>3 x 20 = 60<br>3 x 3 = 9<br>Then:<br><br>60 + 9= 69<br><br>In this step, pupils to use multiplication table knowledge combined with place value understanding to answer the 1s x T1s element of the calculation mentally.</td>
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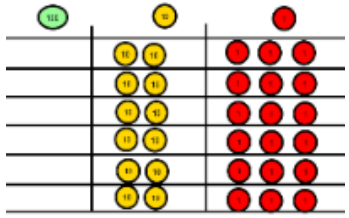
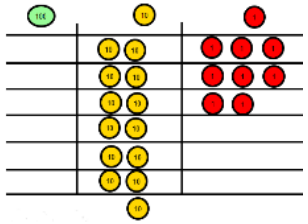
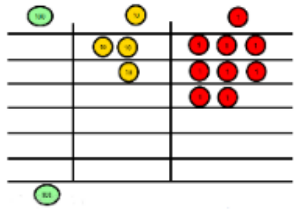
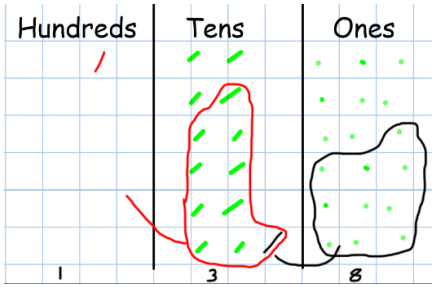
| 4 | **7 – progression through the column method, including involving exchanging.** | Pupils use place value grid and practical equipment, as in stage 6, to support their understanding of the importance of place value in the column method. This understanding is vital for pupils to be able to demonstrate deep understanding of the written method (stage 8).<br><br>Eg, 6 x 23 =<br><br>  <br><u>Step 1:</u> draw the place value grid, and use equipment within the grid to show 6 lots of 23 (NB – note that each lot of 23 is on a separate line).<br><br>  <br><u>Step 2:</u> Focus on the 1s column. 6 x 3 (6 lots of 3, because 3 is in the ones column of 23) = 18. So, an exchange is possible, of ten 1s for one 10. The ten 1s are removed from the 1s column, and a ten is added to the tens column.<br><br>  <br><u>Step 3:</u> Now move to the tens column. 6 lots of 20 is 120, add the ten from step 2 above and that gives 13 tens. So, an exchange is possible, ten tens for one hundred. The ten 10s are removed from the 10s column, and a hundred is added to the hundreds column. | Pupils draw their own place value grids, and draw the practical equipment which they used in the concrete step.<br><br>They make jottings and manipulate their pictorial representations to show the exchanging process.<br><br>  <br> | Supported by their understanding of place value developed in the concrete and pictorial steps of this stage, pupils begin to present their calculations using the written method, which is fully developed during Stage 8 below.<br><br>6 x 23 =<br><br>23<br>× 6<br>138<br>1 1 |

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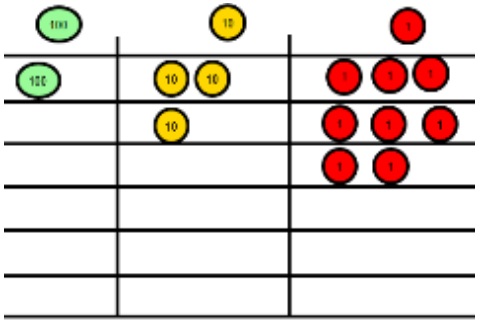
| | | <u>Step 4:</u><br>What digit is shown in each place value column?<br>  | | |
| 5 and 6 | **8 – written method.** | | | Progressing from the abstract step of stage 7, practising the formal written method to achieve fluency in it is a solely abstract step, with distinctive progression within the step.<br><br>Pupils need to be fluent in their understanding of place value in order to carry out the written method with understanding. Furthermore, pupils must have had sufficient practice using equipment then drawings to support their understanding of exchanging.<br><br><u>T1s x 1s</u><br><br>Eg, 56 x 7 =<br>5 6<br>x 7<br>3 9 2<br>4 |

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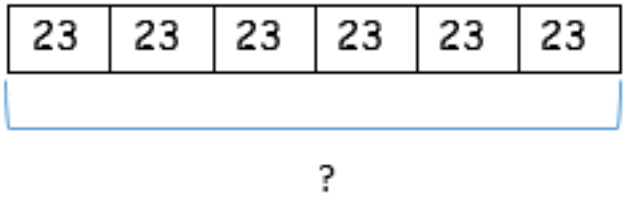
| | | | | <u>T1s x T1s</u><br>The method below can be applied to larger numbers, in addition to multiplying two 2-digit numbers.<br><br>Eg, 56 x 27 =<br><br>5 6<br>x 2 7<br>3 9 4 2 (56 x 7)<br>+ 1 1 1 2 0 (56 x 20)<br>1 5 1 2<br><br><u>Multiplying with decimals</u><br><br>Mastery of decimal place value is dependent on pupils having a fluent understanding of decimal place value.<br><br>Eg,<br><br>3 . 1 9<br>x 8<br>2 5 . 5 2<br>1 7 |

**<u>Developing fluency and mastery of multiplication through variation</u>**

Pupils need to master multiplication as they progress through the stages detailed above. This is achieved through pupils becoming comfortable in presenting their working out in a variety of formats, such as:

Eg:
Different ways to solve, and present solutions to, 6 x 23 =



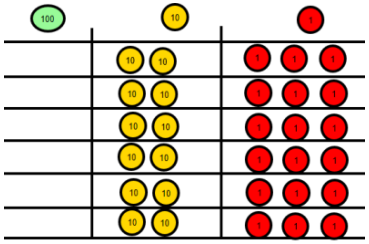

With equipment, prove that 6 x 23 = 138

Explain why 6 x 23 = 23 x 6.

Find the product of 6 and 23.

6 x 23 = ☐

☐ = 6 x 23



Look at this place value grid. What could the calculation be? What is the answer?

<u>Use of worded problems, using the same numbers which lead to the same calculation (6 x 23), but presented in different scenarios:</u>
Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?
Tom saved 23p three days a week. How much did he save in 2 weeks?

Also, give pupils the number sentence and ask them to write a number story (ie worded problem) based on it.

**<u>Prove it and explain questions</u>**
48 x 27 = 1 296. How can you prove this? Questions such as this challenge pupils to show an understanding of multiplication, and encourages pupils to present their proofs in a range of ways (using drawings, bar models, partitioning and so on).

48 x 27 = 1 217. Explain how you know that this is wrong. How do you know from looking at the digit in the 1s place of the answer that the answer is wrong?

Pupils need to be able to explain why they have selected a particular way of presenting their working out.

**Variation is also achieved through varying the language and words which are used to present calculations and questions:**
Groups of, lots of, altogether, equals, count, repeated addition.
Sets of, row, column, multiply, times, ___ times as big as, array.
Array, bar model, number line.
Ten times bigger, 100 times bigger, multiple, product, inverse.
Each.