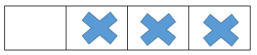
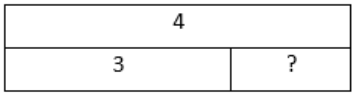
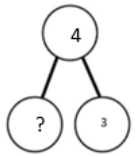
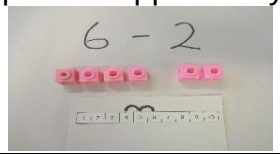
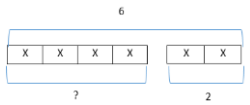
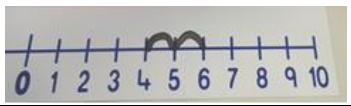
# Subtraction

The table below details the stages that pupils go through in their learning of subtraction, culminating in them carrying out a formal written subtraction method with fluency, accuracy and understanding. The aim is that pupils can identify subtraction calculations for which a mental method is appropriate, but for calculations that they cannot do in their heads they choose an appropriate written method.

Time must be taken building up to the formal written method to ensure complete understanding at each stage.

Each stage has a suggested year group during which the particular stage will be taught. It will not be the case that all pupils in a year group are working at that year group's stage; however, the majority of pupils within a year group will be. Some pupils will need extra time to consolidate their understanding of a stage prior to their own year group's.

Within stages, pupils' learning should start within the Concrete step of the stage (if this is present), with pupils using practical equipment to solve calculations. Learning progresses to the Pictorial step, where pupils draw what they have created in the Concrete step. Finally, pupils should move to the Abstract step. At each stage, a deep understanding (or mastery) of the stage should be demonstrated by pupils. This will be evident through them being able to use the correct method when a calculation is presented in a range of different formats or using varied language, and through pupils explaining the steps involved in the specific calculation method that they are using. The concept of the starting number of a subtraction calculation being the 'whole' is a critical one for pupils to understand at the beginning of their subtraction method learning.

| Year | Stage | Concrete | Pictorial | Abstract |
|---|---|---|---|---|
| 1 | **1 – taking away ones.** | Physically taking away and removing small objects from a whole**.** Pupils physically remove objects (eg multilink, small plastic toys) from the whole.<br>4 – 3 = 1<br>   | Pupils draw the concrete resources (eg multilink) that they have used in the Concrete step, and cross out to show subtraction.<br>  <br>Progress from pupils creating a lifelike drawing of the objects to a bar model to represent the objects:<br>  | Pupils presented subtraction calculations written in a range of formats:<br>4 – 3 =<br>☐ = 4 – 3<br>Pupils taught that 4 is the whole, which is made up of 3 and another number:<br>   |
| 1 and 2 | **2 – counting back.** | Pupils to use practical equipment supported by use of jumps on a numbered number line:<br>  | Pupils draw the Concrete step using a bar model, eg:<br>  | Pupils begin with a numbered line (as in the Concrete step but without use of practical equipment):<br>  |

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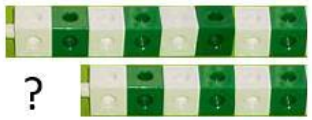
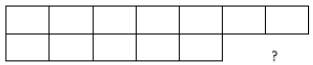
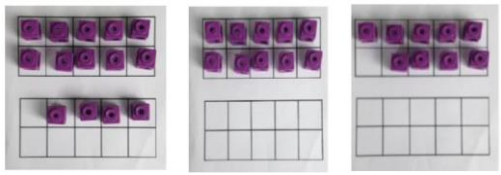
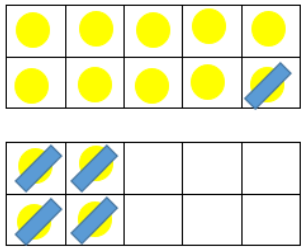
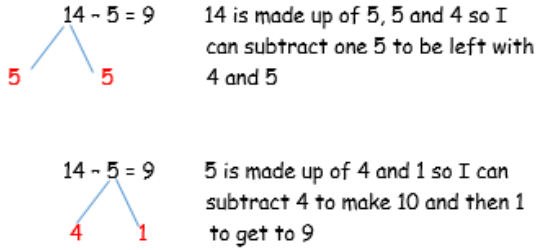
| | | | | Then pupils progress to using blank number lines which they number:<br>  |
| 1 and 2 | **3 – finding the difference.** | Using a range of equipment (eg cubes, numicon and other objects:<br><br>What is the difference between 8 and 6?<br>  <br>  | Pupils to draw the objects that they have used in the Concrete step:<br><br>XXXXXXXX<br>XXXXXX<br><br>Progress to a representation of the objects using a bar model:<br>  | Pupils introduced to a range of vocabulary and language connected with difference:<br><br>Find the difference between 8 and 6.<br><br>8 – 6, the difference is ?<br><br>Children to also explore why 9 - 7 = 8 – 6 (the difference of each digit has changed by 1 ds the difference is the same- this will help when solving 10000-9987). |
| 1 and 2 | **4 – subtraction via making 10.** | This stage is closely related to, and dependent on, pupils acquiring fluency and mastery of number bonds to 10.<br><br>Pupils use a 10 frame with multilink cubes to partition the whole (ie starting number) into 10 and 1s, and subtract the second number.<br>Eg: 14 – 5 =<br>  <br>14 shown as 10 and 4. 4 subtracted to leave 10, and then in the third image another 1 is subtracted to leave 9. | Pupils to draw the 10 frame pictorially:<br><br>14 – 5 =<br>  | Pupils to partition the whole (starting number) into 10s and 1s without the support of the 10 frame or practical equipment.<br>  <br><br>Pupils to develop mastery of subtraction by understanding its anti-commutative nature (ie 14 – 5 does not equal 5 – 14, in contrast with addition, |

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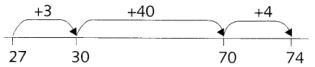
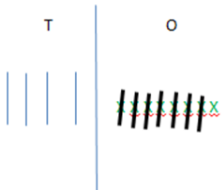
| | | | | which is commutative).<br>Pupils to understand related subtraction facts, eg:<br>14 – 5 = 9 so 14 – 9 = 5. |
| 2 | **5 – use of number lines.** | | Pupils use numbered number lines to count back. This should first be in ones, then tens and then other numbers. | Pupils use empty number lines (onto which they write numbers), or number lines that they have drawn, to count back from the whole (starting number).<br><br>Eg: 74 – 27 =<br>  <br><br>Once pupils are familiar with subtraction as being the inverse of addition they should be taught to use the empty number line to count on.<br>e.g. 74 – 27 = 47<br>  <br><br><br>These steps may be recorded in a different order or combined. With practice, children will record less information and will be able to decide when it is appropriate to count back and count on. |
| 2 | **6 – column method (T1s – 1s).** | Pupils begin with using base 10 equipment.<br>Eg:<br>48 – 7 = and physically remove 7 1s.<br>  | Pupils represent the base 10 equipment pictorially.<br>Eg: 48 – 7 =<br>  <br> | Pupils are introduced to the column method. At this stage the whole (starting number) has a higher value digit in the 1s place than the number being subtracted. |

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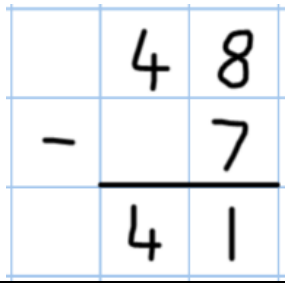
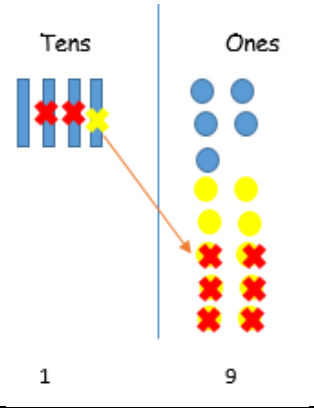
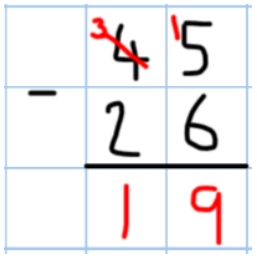
| | | | | Eg: 48 – 7 =<br>  |
| 2 and 3 | **7 – column method with exchanging.** | Pupils begin with using base 10 equipment.<br>Eg: 45 – 26 =<br>  <br>1) Start by partitioning 45<br>2) Exchange one ten for ten ones<br>3) Subtract the ones, then the tens. | Pupils represent pictorially the base 10 used in the Concrete step. At this step, pupils' understanding of the place value columns is vital to support them calculating in the Abstract step.<br>  | Pupils to exchange using the written column method:<br>It is crucial that pupils understand that when they have exchanged the 10 they still have 45. 45 = 30 + 15<br>  |

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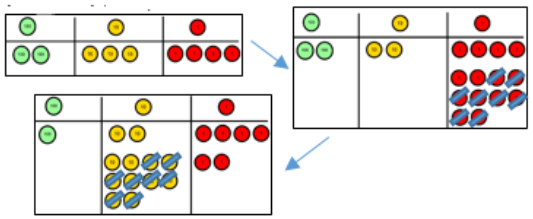
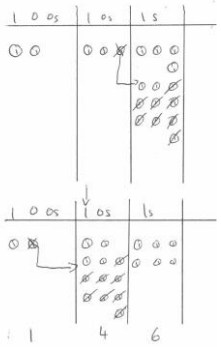
| 4 and 5 | **8 – expanded column method, initially using counters within place value grids.** | Pupils to use counters (eg multilink) within drawn place value grids (or pupils to draw their own place value grids and place multilink cube on them).<br>Eg: 234 – 88 =<br>  <br>Pupils use the multilink to physically exchange between the place value columns. | Pupils to be taught to represent the counters pictorially, using a place value grid that they have drawn.<br>Eg: 234 – 88 =<br>  | Rather than drawing counters, pupils partition each number of the calculation using place value columns:<br>Eg: 234 – 88 =<br>H T 1s<br>200 30 4<br>- 80 8<br><br>Pupils check that the upper number in each place value column is greater than the lower number. Starting with the ones column,<br>pupils realise that they cannot take 4 ones from 8 ones so they exchange a ten for ten ones:<br><br>H T 1s<br>20<br>200 ~~30~~ 14<br>- 80 8<br><br>Pupils then move to check the tens column, and exchange if necessary:<br>H T 1s<br>100 120<br>~~200~~ ~~30~~ 14<br>- 80 8 |

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| | | | | Pupils then carry out the subtraction in each column, beginning with the 1s column:<br>H T 1s<br>100 120<br>~~200~~ ~~30~~ 14<br>- 80 8<br>100 40 6<br><br>Pupils then recombine each place value column to give their final answer, in this example 146. |
| 4, 5 and 6 | **9 – compact column method.** | | | This is the formal standard method of subtraction. This should only be taught when pupils are completely confident in using place value columns, exchanging and the expanded column method.<br><br>Eg:<br>74 – 27 = 47<br><br>$^{6}\not{7}^{1}4$<br>−2 7<br>4 7<br><br>563 – 271 = 292<br><br>$^{4}\not{5}^{1}6\ 3$<br>− 2 7 1<br>2 9 2<br><br>It is important that pupils say and hear “16 tens take away 7 tens” not “16 -7”. Use the term “exchange” not borrow. |

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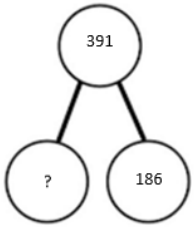
| | | | | Pupils should label each place value column if this aids their transition from the expanded method to the compact method.<br><br>Within this stage, pupils should progress through the following, using the compact method:<br><br>1. Subtraction of a two digit number, no exchange required.<br>2. Subtraction of a two digit number, exchange required.<br>3. Subtraction of a 3 digit number, one lot of exchange required.<br>4. Subtraction of a three digit number, 2 lots of exchange required.<br>5. Subtraction involving decimal numbers with up to 3 decimal places in a variety of contexts, including money.<br>6. Subtraction of decimal numbers from whole numbers, eg: 6 – 2.3145 =, where a decimal point and zeros have to be written to the right of the whole number within the column. |

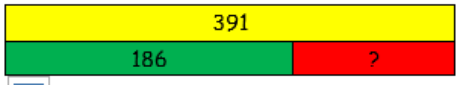
## Developing fluency and mastery of subtraction through variation

Pupils need to master subtraction as they progress through the stages detailed above. This is achieved through pupils becoming comfortable in presenting their working out in a variety of formats, such as:

Eg: 391 – 186 =




or



Pupils need to be able to explain why they have selected a particular way of presenting their working out.

**Variation is also achieved through varying the language and words which are used to present calculations and questions:**

Raj spent £391, Timmy spent £186. **How much more** did Raj spend?
I had 391 metres to run. After 186 metres I stopped. How many metres do I have **remaining** to run?
Kirsty scores 391 points in a quiz. Sam scores 186. What is the **difference** in their scores?

These questions could be answered in a range of ways, including pupils drawing a visualisation of the subtraction using the bar model above.

It is vital that language is varied so that pupils hear and see a range of ways of asking subtraction questions.

Questions need to be presented in a variety of ways so that pupils develop their understanding of, for example, the = sign meaning 'equals' rather than 'makes'.
Eg,
391 - 186 = ☐

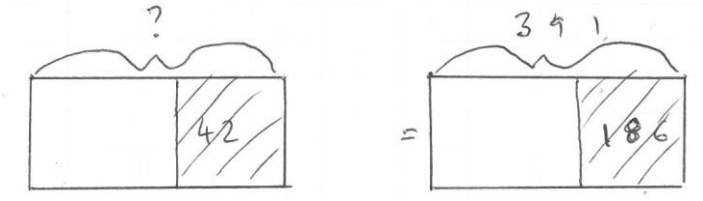
Presented as:

☐ = 391 – 186

This can be progressed to:

☐ - 42 = 391 – 186

As explained above, pupils should learn about a range of strategies to solve these style of questions, including use of bar modelling:



**Key words which pupils need to become used to hearing and seeing when learning subtraction:**

Take, take away, less, fewer, how many left, equals.
Partition, difference, count back, count on, less than, subtract, minus, tens, ones, place value, place value column.
Exchange, decrease, tens, ones.
Inverse, tenths, hundredths, thousandths, decimal point, decimal.

**Prove it and explain questions**

161 – 28 = 133. How can you prove this? Questions such as this challenge pupils to show an understanding of subtraction as being the inverse of addition, as well encouraging pupils to present their proofs in a range of ways (using drawings, bar models, partitioning).

441 - 126 = 567. Explain how you know that this is wrong. Questions such as this require pupils to understand the nature of subtraction: specifically that subtraction of positive numbers decreases value.